Grade 3 Mathematics Standards

Critical Areas for COHERENCE in Mathematics in Grade 3

In Grade 3, instructional time should focus on five critical areas:

1. Developing an understanding of all operations with a focus on multiplication and division and strategies for multiplication and division within 100. Students develop and refine their understanding of all operations to solve multistep problems and focus on the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations (e.g., Associative Property and Distributive Property) to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division. Students understand that a word problem can be represented with an equation based on the situation, but the solution may use a related equation that is easier to manipulate (e.g., a word problem may be represented with a situation equation such as 54+? = 78; and students understand that even though the word problem is a joining situation, it is easier to solve using a subtraction equation {78 − 54 =? }).

2. Developing understanding of fractions, especially unit fractions (fractions with numerator of 1). Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. Developing understanding of the structure of rectangular arrays and of area. Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

Standards for Mathematical Practice in Grade 3

The State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Grade 3 students complete.

Practice	Explanation and Example
1) Make Sense and Persevere in Solving Problems.	Mathematically proficient students in Grade 3 examine problems, can make sense of the meaning of the task, and find an entry point or a way to start the task. Grade 3 students also develop a foundation for problem solving strategies and become independently proficient on using those strategies to solve new tasks. They might use concrete objects or pictures to show the actions of a problem. If students are not at first making sense of a problem or seeing a way to begin, they ask questions that will help them get started. They are expected to persevere while solving tasks; that is, if students reach a point in which they are stuck, they can reexamine the task in a different way and continue to solve the task. Students in Grade 3 complete a task by asking themselves the question, “Does my answer make sense?” Example: to solve a problem involving multi-digit numbers, they might first consider similar problems that involve multiples of ten or one hundred. Once they have a solution they look back at the problem to determine if the solution is reasonable and accurate. They often check their answers to problems using a different method or approach.
2) Reason abstractly and quantitatively.	Mathematically proficient students in Grade 3 recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of the quantities. This involved two processes: decontextualizing and contextualizing. In Grade 3, students represent situations by decontextualizing tasks into numbers and symbols. For example, to find the area of the floor of a rectangular room that measures 10 ft by 12 ft, a student might represent the problem as an equation, solve it mentally, and record the problem and solution as 10 x 12 = 120 ft squared. She has decontextualized the problem. When she states at the end that the area of the room is 120 square feet, she has contextualized the answer in order to solve the original problem. Problems like this that begin with a context and are then represented with mathematical objects or symbols are also examples of modeling with mathematics (SMP 4).
3) Construct viable arguments and critique the reasoning of others.	Mathematically proficient students in Grade 3 accurately use definitions and previously established solutions to construct viable arguments about mathematics. Grade 3 students might construct arguments using concrete referents such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking. For example, when comparing the unit fractions 13$aaaaaa$16 students may generate their own representation of both fractions and then critique each other’s reasoning in class, as they connect their arguments to the representations that they created. Students in Grade 3 present their arguments in

Practice	Explanation and Example
	the form of representations, actions on those representations, and explanations in words (oral and written).
4) Model with mathematics.	Mathematically proficient students in Grade 3 experiment with representing problem situations in multiple ways, including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. They model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. Students should have ample opportunities to connect the different representations and explain the connections. Grade 3 students should evaluate their results in the context of the situation and reflect on whether the results make sense.
5) Use appropriate tools strategically.	Mathematically proficient students in Grade 3 consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. The tools that students in Grade 3 might use physical objects (place value (base ten) blocks, hundreds charts, number lines, tape diagrams, fraction bars, arrays or area models, tables, graphs, and concrete geometric shapes (e.g. pattern blocks, 3-d solids) paper and pencil, rulers and other measuring tools, grid paper, virtual manipulatives, and concrete geometric shapes (e.g., pattern blocks, 3-d solids), etc. Students should also have experiences with educational technologies, such as calculators and virtual manipulatives that support conceptual understanding and higher-order thinking skills. During classroom instruction, students should have access to various mathematical tools as well as paper, and determine which tools are the most appropriate to use. For example, when comparing 16$aaaaaa$34, students can use benchmark fractions and the number line to reason and explain that 34would be placed to the right of 12 because it is a "a little more than 12 or they might say " 34 $iiii$ 14 away from 1 whole". When students model situations with mathematics, they are choosing tools appropriately (SMP 5). As they decontextualize the situation and represent it mathematically, they are also reasoning abstractly (SMP2).
6) Attend to precision.	Mathematically proficient students in Grade 3 are precise in their communication, calculations, and measurements. In all mathematical tasks, they communicate clearly, using grade-level appropriate vocabulary accurately as well as giving precise explanations and reasoning regarding their process of finding solutions. For example, while measuring objects iteratively (repetitively), students check to make sure that there are no gaps or overlaps. In using representations, such as pictures, tables, graphs, or diagrams, they use appropriate labels to communicate the meaning of their representation. During tasks involving number sense, students check their work to ensure the accuracy and reasonableness of solutions.
7) Look for and make use of structure.	Mathematically proficient students in Grade 3 carefully look for patterns and structures in the number system and other areas of mathematics. Grade 3 students use structures such as place value, the properties of operations, other generalizations about the behavior of the operations (for example, the less you subtract, the greater the difference), and attributes of shapes to solve problems. In many cases, they have identified and described these structures through repeated reasoning (SMP 8). For example, when Grade 3 students calculate 16 X 9, they might apply the structure of

Practice	Explanation and Example
	place value and the distributive property to find the product: 16×9=(10+6)×9=(10×9)+(6×9). Students in Grade 3 should be using and explaining how they are using the different properties of operations to solve problems.
8) Look for and express regularity in repeated reasoning.	Mathematically proficient students in Grade 3 notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7×8, they might decompose 7 into 5 and 2 and then multiply 5×8 and 2×8 to arrive at 40+16 or 56. Mathematically proficient 3rd graders formulate conjectures about what they notice. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Mathematics Content Standards in Grade 3

Operations and Algebraic Thinking 3.OA

(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 22)

Represent and solve problems involving multiplication and division.
(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 22)

3.OA.1. Interpret products of whole numbers*, (e.g. interpret* $5 \cdot 7$ *as the total number of objects in 5 groups of 7 objects each*.)

3.OA.2. Interpret whole-number quotients of whole numbers, *(e.g. interpret* $56 \div 8$ *as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.)*

3.OA.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays*,* and measurement quantities*, (e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.)* Refer to shaded section of Table 2 for specific situation types.

3.OA.4. Determine the unknown whole number in a multiplication or division equation by using related equations. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \cdot ? = 48$; $5 = \blacksquare \div 3$; $6 X 6 = \_\_$

Understand properties *of* multiplication and the relationship between multiplication and division.
(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 24)

3.OA.5. Apply properties of operations as strategies to multiply and divide. *Examples: If* $6 \cdot 4 = 24$ *is known, then* $4 \cdot 6 = 24$ *is also known. (Commutative property of multiplication.)* $3 \cdot 5 \cdot 2$ *can be found by* $3 \cdot 5 = 15$*, then* $15 \cdot 2 = 30$ *, or by* $5 \cdot 2 = 10$ *, then* $3 \cdot 10 = 30$ *. (Associative property of multiplication.) Knowing that* $8 \cdot 5 = 40$ *and* $8 \cdot 2 = 16$ *, one can find* $8 \cdot 7$ *as* $8 \cdot (5 + 2) = (8 \cdot 5) + (8 \cdot 2) = 40 + 16 = 56$*. (Distributive property.) Students need not use formal terms for these properties.*

3.OA.6. Understand division as an unknown-factor problem. *For example, find* $32 \div 8$ *by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100 (basic facts up to 10 x 10).

3.OA.7. Fluently (efficiently, accurately, and flexibly) multiply and divide with single digit multiplications and related divisions using strategies *(e.g. relationship between multiplication and division, doubles, double and double again, half and then double, etc.)* or properties of operations.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.
(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 27 Paragraph 2)

3.OA.8. Solve two-step word problems using any of the four operations. Represent these problems using both situation equations and/or solution equations with a letter or symbol standing for the unknown quantity (*refer to* Table 1 *and* Table 2 *and standard* 3.OA.3). Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole-number answers.

For Example:
A clown had 20 balloons. He sold some and has 12 left. Each balloon costs $2. How much money did he make?
Situation Equation: 20 – n = 12
n x $2 = ☐
Solution Equation: 20 -12 = n
n x $2 = ☐

3.OA.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations (See Table 5). *For example, observe that 4 times a number is always even, and explain why 4 times a number can be* ***decomposed*** *into two equal addends.*

Number and Operations in Base Ten 3.NBT

(Numbers & Operations Base 10 Progression K-5 Pg. 12)

Use place value understanding and properties of operations to perform multi-digit arithmetic.

3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2. Fluently (efficiently, accurately, & flexibly) add and subtract within 1000 using strategies *(e.g. composing/decomposing by like base-10 units, using friendly or benchmark numbers, using related equations, compensation, number line, etc.)* and algorithms (including, but not limited to: traditional, partial-sums, etc.) based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.NBT.3. Multiply one-digit whole numbers by multiples of 10 in the range 10 to 90 *(e.g.* $9 \cdot 80$*,* $5 \cdot 60$) using strategies based on place value and properties of operations.

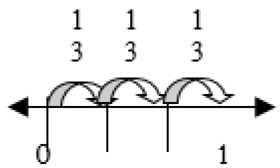
Number and Operations—Fractions 3.NF

Develop understanding of fractions as numbers.
(Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)
(Number and Operations – Fractions Progression Pg. 3-5)

3.NF.1. Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by *a* parts of size $\frac{1}{b}$.

3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3.NF.2a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

Ex: 

3.NF.2b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line (a is the countable units of $\frac{1}{b}$ that determines the place on the number line).

3.NF.3. Explain **equivalence** of fractions, and compare fractions by reasoning about their size (it is a mathematical convention that when comparing fractions, the whole is the same size).

3.NF.3a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

3.NF.3b. Recognize and generate simple equivalent fractions, *(e.g.* $\frac{1}{2} = \frac{2}{4}, \frac{4}{6} = \frac{2}{3}$*.)* Explain why the fractions are equivalent, *e.g. by using a visual fraction model*.

3.NF.3c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form* $3 = \frac{3}{1}$*; recognize that* $\frac{6}{1} = 6$*; locate* $\frac{4}{4}$ *and 1 at the same point of a number line diagram.*

3.NF.3d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the relational symbols >, <, =, or ≠, and justify the conclusions, *(e.g. by using a visual fraction model.)*

Measurement and Data 3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1. Tell and write time to the nearest minute using a.m. and p.m. and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, *(e.g. by representing the problem on a number line diagram.)* (See Table 1)

3.MD.2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l) (Excludes cubed units such as cm^3 and finding the geometric volume of a container).

3.MD.3. Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, *(e.g. by using drawings (such as a beaker with a measurement* *scale) to represent the problem.)* (Excludes multiplicative comparison problems) (See Table 1 and Table 2).

Represent and interpret data.

3.MD.4. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. (See Table 1). *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* (Measurement and Data (data part) Progression K–5 Pg. 7)

3.MD.5. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. (Measurement and Data (data part) Progression K–5 Pg. 10)

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.6. Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.MD.6a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area (does not require standard square units).

3.MD.6b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units (does not require standard square units).

3.MD.7. Measure areas by counting unit squares (square cm, square m, square in, square ft, and non-standard square units).

3.MD.8. Relate area to the operations of multiplication and addition (Measurement and Data (measurement part) Progression K–5 Pg. 16).

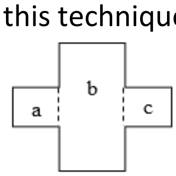
3.MD.8a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

3.MD.8b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

3.MD.8c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \cdot b$ and $a \cdot c$. Use area models to represent the distributive property in mathematical reasoning (Supports 3.OA.5).

(Measurement and Data (measurement part) Progression K–5 Pg. 18).

3.MD.8d. Recognize area as additive. Find areas of **rectilinear** figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. *Example:*



Students can find the total area of the shape by finding the areas of a, b, and c and adding them together.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.9. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. (Measurement and Data (measurement part) Progression K–5 Pg. 16)

Geometry 3.G

Reason with shapes and their attributes.

(Geometry Progression K-6 Pg. 13)

3.G.1. Understand that shapes in different categories *(e.g. rhombuses, rectangles,* **trapezoids***, kites and others)* may share attributes *(e.g. having four sides),* and that the shared attributes can define a larger category *(e.g. quadrilaterals).* Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. Refer to inclusive definitions noted in the glossary.

3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as* $\frac{1}{4}$ *of the area of the shape.*